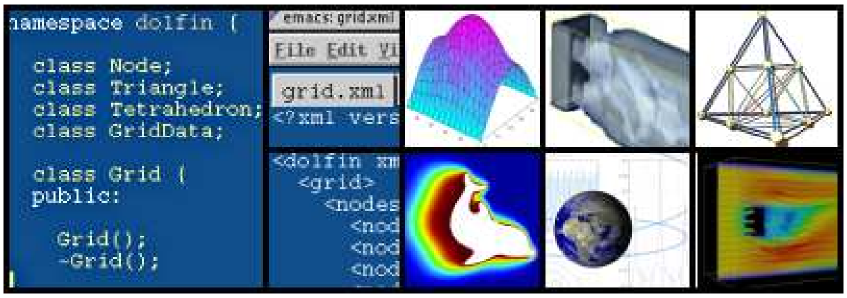
PDE Project Course 05/06

Suggestions for projects



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General guidelines

This document contains a list of projects. Since these are only suggestions, you are welcome with your own ideas. Regard the list as an inspiration, and perhaps a hint on the expected level of your projects.

Concerning grades, the projects are divided into two parts: basic level and advanced level. Basic level means grade 3 and advanced level means grade 4 or 5. However, advanced level is no guarantee for grade 4 or 5. It is also required that your report and your presentation match the level of your project. It is also possible to receive a higher grade even if you only complete the basic level, if you deliver an excellent report and an excellent presentation.

Good luck!
Johan

1 Convection-Diffusion

Implement your own solver for the convection-diffusion equation in 2 dimensions using Fenics/DOLFIN. Implement streamline diffusion stabilization and verify that it performs as expected.

Advanced

Extend your solver to 3D.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

2 Chemical reactions

Simulate the following system of chemical reactions, where the substances A and B react to form C: $A + B \to C$.

Consider a beaker containing a solution of A with given concentration. To this beaker, we add a drop of B every second until finally A has "completely" reacted with B. Try to find a suitable reaction to simulate in a chemistry book. Maybe the reaction you want to simulate is instead given by $2A+3B \to 4C$, or perhaps $5A + 2B + C \to 2C$?

Model this as a system of reaction–diffusion equations, where $u_1(x, t)$ and $u_2(x, t)$ are the two concentrations to be determined.

Implement your 2D solver using Fenics/DOLFIN.

Advanced

Solve the problem in 3D.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

3. Some suitable book on chemistry.

3 Elasticity

Extend the existing linear elasticity module in DOLFIN to also consider anisotropic materials (different stiffness in different directions).

Advanced

Apply this extension also to the existing elasticity module for large deformations.

References

1. *Beyond the Elements of Finite Elements: General Principles for Solid and Fluid Mechanics Applications* by Hansbo. Department of Solid Mechanics, Chalmers University of Technology, 2002

4 Error estimation/adaptivity

Write a solver for the convection-diffusion equation (or choose your own equation) using DOLFIN. Compute error estimates using duality and study how the solution changes when the mesh size h is changed.

Advanced

Consider one of the following extensions:

- Compute error estimates and refine the mesh manually where the error is large. Study the convergence of the solution.
- Also study convergence w.r.t. the order p.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.
2. *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

5 Bistable equation

Write a solver in Dolfin for the bistable equation in 2D, which is an easy example of a nonlinear PDE.

Advanced

Extend the solver to 3D.

References

1. *Body and Soul computer sessions (Reaction-Diffusion)*: `http://www.phi.chalmers.se/bodysoul/sessions/`.

6 The Navier-Stokes equations

Extend the existing Navier-Stokes module in DOLFIN with variable density. This involves adding an extra equation for the density ρ:

$$\begin{pmatrix} \dot{(\rho v)} + v \cdot \nabla(\rho v) - \nu \Delta v + \nabla p \\ \nabla \cdot (\rho v) \\ \dot{\rho} + \nabla \cdot (\rho v) \end{pmatrix} = \begin{pmatrix} f \\ 0 \\ 0 \end{pmatrix}$$

Advanced

Nothing extra is needed for advanced level.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Differential Equations* by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

3. *Adaptive finite element methods for turbulent flow* by Johan Hoffman. Chalmers Finite Element Center Preprint 2002–08, available at `http://www.phi.chalmers.se/preprints/`.

7 The compressible Euler equations

Implement a solver for the compressible Euler equations in DOLFIN (very advanced). See the paper on compressible flow in the literature list on the course page.

Advanced

Nothing extra is needed for advanced level.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Differential Equations* by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

3. *Adaptive finite element methods for turbulent flow* by Johan Hoffman. Chalmers Finite Element Center Preprint 2002–08, available at `http://www.phi.chalmers.se/preprints/`.

8 Maxwell's equations

Implement a solver for Maxwell's equations in DOLFIN (fairly advanced). It will be necessary to implement edge elements (Nedelec) to achieve this. Ask the teacher for material on computational electromagnetics if you are interested in such a project.

Advanced

Nothing extra is needed for advanced level.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Electromagnetics* by Bondeson, Rylander and Ingelström. Springer 2005.