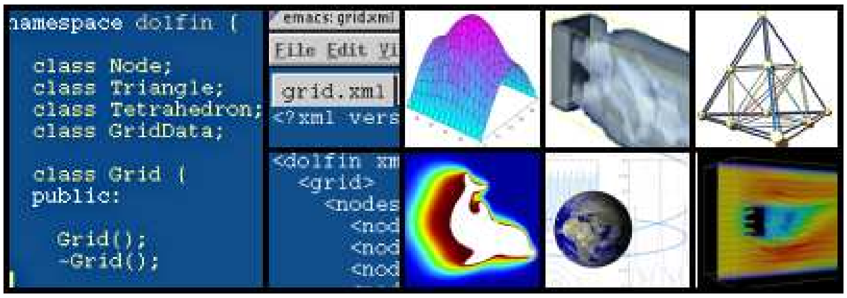
PDE Project Course 04/05

Suggestions for projects



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General guidelines

This document contains a list of projects. Since these are only suggestions, you are welcome with your own ideas. Regard the list as an inspiration, and perhaps a hint on the expected level of your projects.

Concerning grades, the projects are divided into two parts: basic level and advanced level. Basic level means grade 3 and advanced level means grade 4 or 5. However, advanced level is no guarantee for grade 4 or 5. It is also required that your report and your presentation match the level of your project. It is also possible to receive a higher grade even if you only complete the basic level, if you deliver an excellent report and an excellent presentation.

Good luck!
Johan and Karin

1 Convection-Diffusion

Implement your own solver for the convection-diffusion equation in 2 dimensions from scratch in Matlab.

Advanced

Extend your solver to 3D.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

2 Chemical reactions

Simulate the following system of chemical reactions, where the substances A and B react to form C: $A + B \to C$.

Consider a beaker containing a solution of A with given concentration. To this beaker, we add a drop of B every second until finally A has "completely" reacted with B. Try to find a suitable reaction to simulate in a chemistry book. Maybe the reaction you want to simulate is instead given by $2A+3B \to 4C$, or perhaps $5A + 2B + C \to 2C$?

Model this as a system of reaction–diffusion equations, where $u_1(x,t)$ and $u_2(x,t)$ are the two concentrations to be determined.

Implement your 2D solver as a module in DOLFIN.

Advanced

Solve the problem in 3D in Dolfin.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

3. Some suitable book on chemistry.

3 Heat equation

Write a solver for the heat equation in 2D from scratch (in Matlab). Compute error estimates (energy norm and/or L_2-norm) and study how the solution changes when the mesh size h is changed.

Advanced

Consider one of the following extensions:

- Do the computations in 3D.
- Compute error estimates and refine the mesh manually where the error is large. Study the convergence of the solution.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.
2. *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

4 The Navier-Stokes equations

Implement a solver for the Navier-Stokes equations in 2D or 3D.

Advanced

Nothing extra is needed for advanced level.

References

1. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.

2. *Computational Differential Equations* by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996.

3. *Adaptive finite element methods for turbulent flow* by Johan Hoffman. Chalmers Finite Element Center Preprint 2002–08, available at `http://www.phi.chalmers.se/preprints/`.

5 Elasticity

Implement a solver for linear elasticity in Puffin in 2D. Assume that your materials are isotropic (same stiffness in all directions).

Advanced

Extend the solver to also handle anisotropic materials.

References

1. *Beyond the Elements of Finite Elements: General Principles for Solid and Fluid Mechanics Applications* by Hansbo. Department of Solid Mechanics, Chalmers University of Technology, 2002

6 The Wave equation

Read the technical report by L.Beilina and implement the method in 2D.

Advanced

Implement the absorbing boundary condition.

References

1. *A Hybrid Method for the Wave Equation*, by L. Beilina, Technical report, Chalmers Finite Element Center (2001)
2. *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003.
3. *Computational Differential Equations* by Eriksson, Estep, Hansbo and Johnson. Studentlitteratur 1996.

7 Bistable equation

Write a solver in Dolfin for the bistable equation in 2D, which is an easy example of a nonlinear PDE.

Advanced

Extend the solver to 3D.

References

1. *Body and Soul computer sessions (Reaction-Diffusion)*: `http://www.phi.chalmers.se/bodysoul/sessions/`.