PDE Project Course

Introduction

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Welcome

Teachers:

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Course page:
Follow the links education $\rightarrow$ courses, starting at

- `http://www.math.chalmers.se/cm/`

Partial differential equations (PDE)

Solve

$$A(u) = f$$

where A is a differential operator, f is a given force term and u is the solution.

Important questions:

- Existence/uniqueness of solutions
- Computation of solutions

Contents of the course

- Computation of solutions to PDEs
- Finite Element Method
- Mostly implementation, not so much theory
- Programming (C++ or Matlab)
- Independent work
- Presentations of results and report writing

Prerequisites

- PDE/FEM course
- Some knowledge of either C++ or Matlab (excellent opportunity to learn some C++)

Examples of PDE

- $A(u) = -\Delta u = f$ Poisson's equation
- $A(u) = \dot{u} - \Delta u = f$ The heat equation
- $A(u) = \ddot{u} - \Delta u = f$ The wave equation

These are the main examples of linear *elliptic*, *parabolic*, and *hyperbolic* PDE.

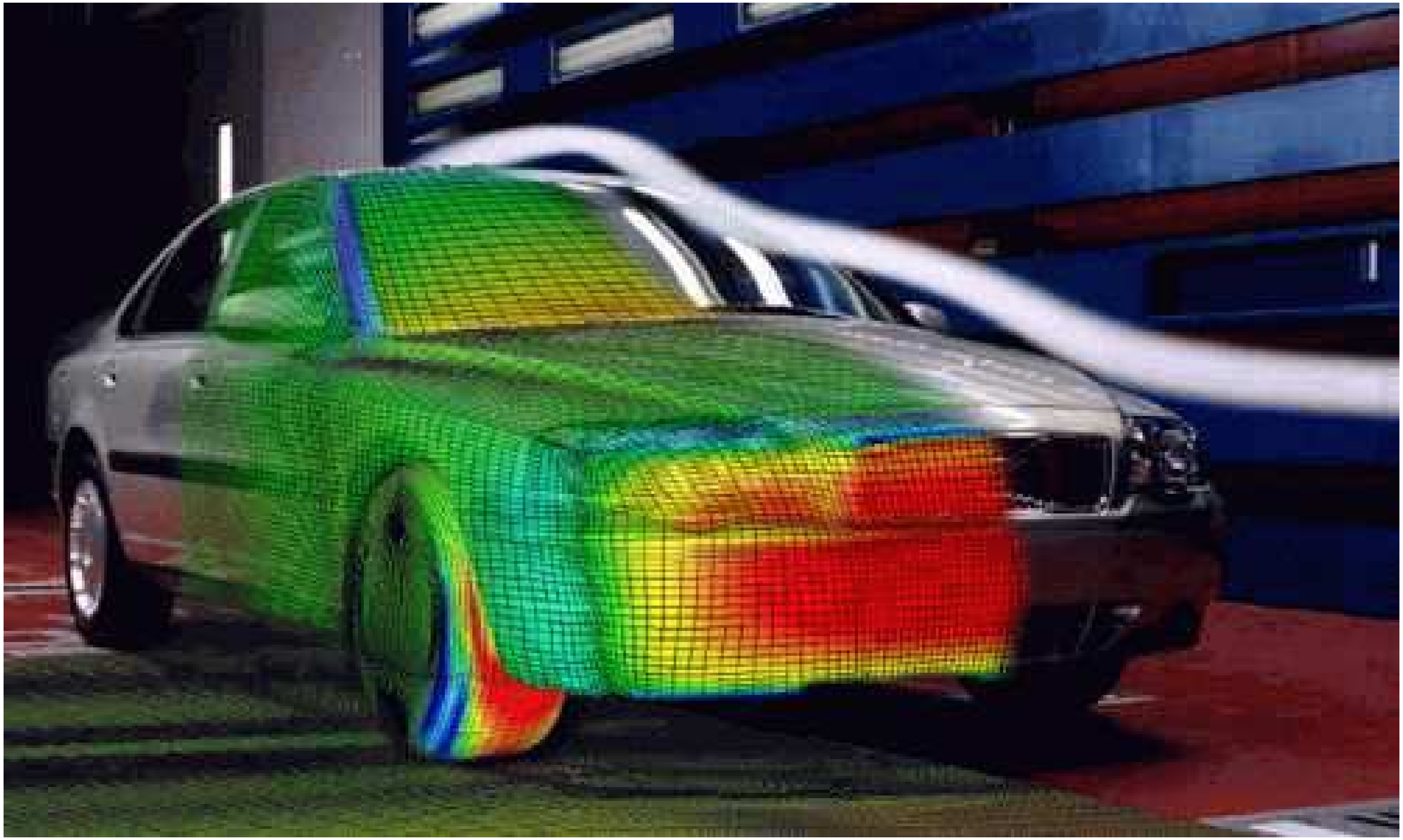
Examples of PDE

The Navier–Stokes equations:

$$A(u) = \begin{pmatrix} \dot{v} + v \cdot \nabla v - \nu \Delta v + \nabla p \\ \nabla \cdot v \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}$$

where the solution $u = (v, p)$ consists of the the fluid velocity v and the pressure p.

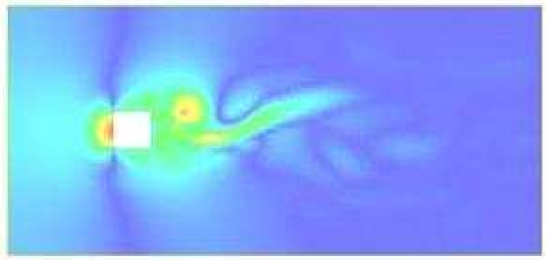
Examples of PDE



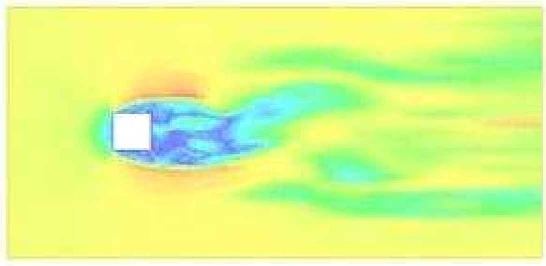
Numerical solution of the Navier-Stokes equations.

Examples of PDE

Turbulent flow around a surface mounted cube:
Pressure:



Norm of velocity:



Examples of PDE

The equations of linear elasticity:

$$
\begin{aligned}
\frac{\partial u}{\partial t} - v = 0 \quad &\text{in } \Omega^0, \\
\frac{\partial v}{\partial t} - \nabla \cdot \sigma = f \quad &\text{in } \Omega^0, \\
\sigma = E\epsilon(u) = E(\nabla u^\top + \nabla u)& \\
v(0,\cdot) = v^0, \quad u(0,\cdot) = u^0 \quad &\text{in } \Omega^0.
\end{aligned}
$$

Only valid for small displacements.

Examples of PDE

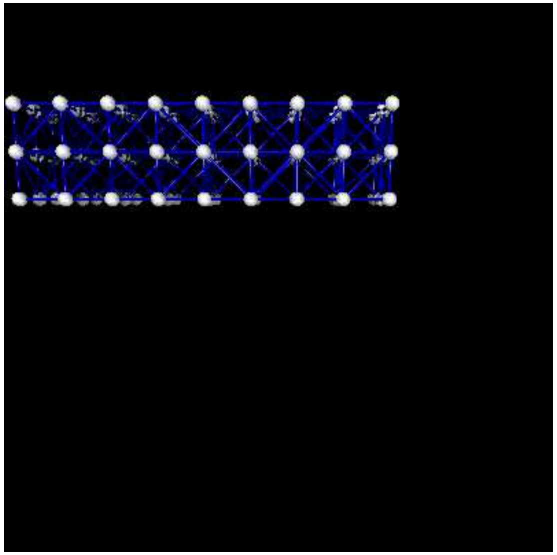
Updated Lagrangian formulation:

$$
\begin{aligned}
\frac{\partial u}{\partial t} - v = 0 \quad \text{in } \Omega(t), \\
\frac{\partial v}{\partial t} - \nabla \cdot \sigma = f \quad \text{in } \Omega(t), \\
\frac{\partial \sigma}{\partial t} = E\epsilon(v) = E(\nabla v^\top + \nabla v) \\
v(0,\cdot) = v^0, \quad u(0,\cdot) = u^0 \quad \text{in } \Omega^0.
\end{aligned}
$$

Also valid for large displacements.

Examples of PDE

Elastic beam (rubber) under gravity:



Literature

PDE/FEM:

- *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003
- *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996

Books and material

Programming:

- *C++ Primer*, by Lippman. Addison-Wesley 1995 (Old but quite good)
- *C++ direkt*, by Skansholm. Studentlitteratur 2000 (Only in swedish)
- *The C++ Programming Language*, by Stroustrup. Addison-Wesley 1997 (Not for beginners)

Resources on the web

- *Body and Soul computer sessions*: http://www.phi.chalmers.se/bodysoul/sessions/

Software Tools

- Write your own solver
- Puffin (MatLab/Octave)
- Dolfin (C++)
- FemLab (mesh generation/visualisation)
- OpenDX (visualisation in 3D)
- Triangle (mesh generation in 2D)
- TetGen (mesh generation in 3D)

Lecture plan

1. *Finite element method*
2. *Implementation of finite element methods*
3. *(C++ programming)*
4. *An introduction to DOLFIN and Puffin*

Schedule

For a detailed schedule look at the homepage.
Lectures:

- Tuesdays 10-12 in S1
- Fridays 10-12 in S4

Consultation:

- e-mail preferred
- Tuesdays 10-12
- Fridays 10-12

Deadlines

Project plan:

- Friday January 28 at 17.00

Progress reports:

- course week 4 and 6

Presentations:

- course week 9

Project report:

- Friday March 18 at 17.00

DOLFIN

- Numerical solution of PDE using FEM
- 2D / 3D
- Object-oriented (C++)
- GPL licence

Puffin

- Numerical solution of PDE using FEM
- 2D
- Written for Matlab (Octave)
- GPL licence

Project / Examination

- Groups of max 2 students
- Submit a project plan
- Two mandatory oral progress reports
- Presentation
- Written report

Project plan

- Easy PDE - write solver from scratch
- Advanced PDE - use existing tools (if you want)
- State what grade the group is aiming for

Project ideas

- ...