PDE Project Course

4. An Introduction to DOLFIN and Puffin

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Lecture plan

- **DOLFIN**
  - Overview
  - Input / output
  - Using DOLFIN
  - Summary of features
- **Puffin**
  - Overview
  - Using Puffin

Overview of DOLFIN

Introduction

- An adaptive finite element solver for PDEs and ODEs
- Developed at the Department of Computational Mathematics
- Written in C++
- Only a solver. No mesh generation. No visualization.
- Licensed under the GNU GPL
- `http://www.phi.chalmers.se/dolfin`

Evolution of DOLFIN

- First public version, 0.2.6, released Feb 2002
- The latest version, 0.4.5, released Feb 2004
- Latest version consists of 41.000 lines of code

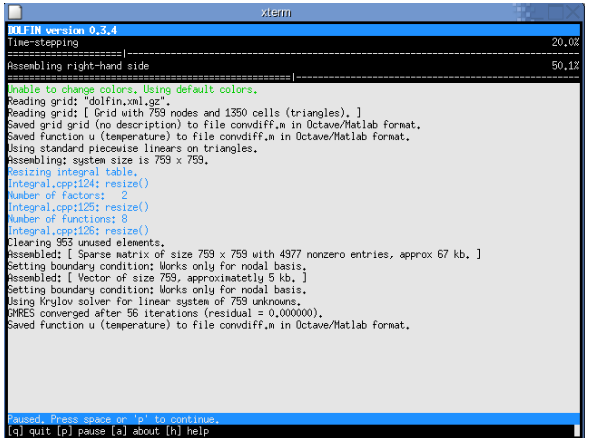
GNU and the GPL

- Makes the software free for all users
- Free to modify, change, copy, redistribute
- Derived work must also use the GPL license
- Enables sharing of code
- Simplifies distribution of the program
- Linux is distributed under the GPL license
- See `http://www.gnu.org`

Features

- 2D or 3D
- Automatic assembling
- Triangles or tetrahedrons
- Linear elements
- Algebraic solvers: LU, GMRES, CG, preconditioners

Everyone loves a screenshot



Examples

- *Start movie 1 (driven cavity, solution)*
- *Start movie 2 (driven cavity, dual)*
- *Start movie 3 (driven cavity, dual)*
- *Start movie 4 (bluff body, solution)*
- *Start movie 5 (bluff body, dual)*
- *Start movie 6 (jet, solution)*
- *Start movie 7 (transition to turbulence)*

Input / output

Input / output

- OpenDX: free open-source visualization program based on IBM:s *Visualization Data Explorer*.
- MATLAB: commercial software (2000 Euros)
- GiD: commercial software (570 Euros)

Note: Input / output has been redesigned in the 0.3-4.x versions of DOLFIN and support

has not yet been added for OpenDX and GiD.

GiD / MATLAB

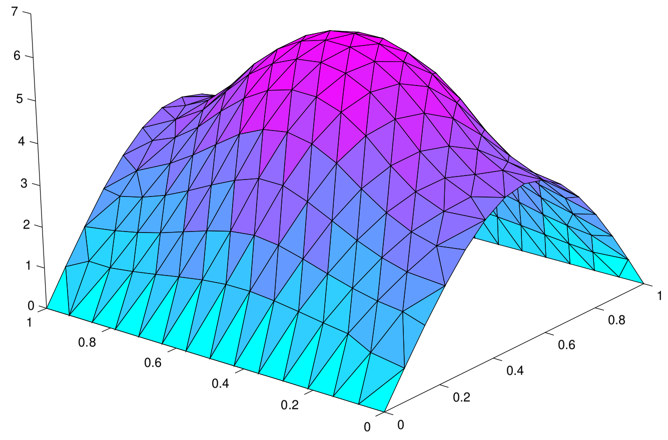
Poisson's equation:

$$-\Delta u(x) = f(x), \quad x \in \Omega,$$

on the unit square $\Omega = (0,1) \times (0,1)$ with the source term f localised to the middle of the domain.

Mesh generation with **GiD** and visualization using the `pdesurf` command in **MATLAB**.

GiD / MATLAB



MATLAB / GiD

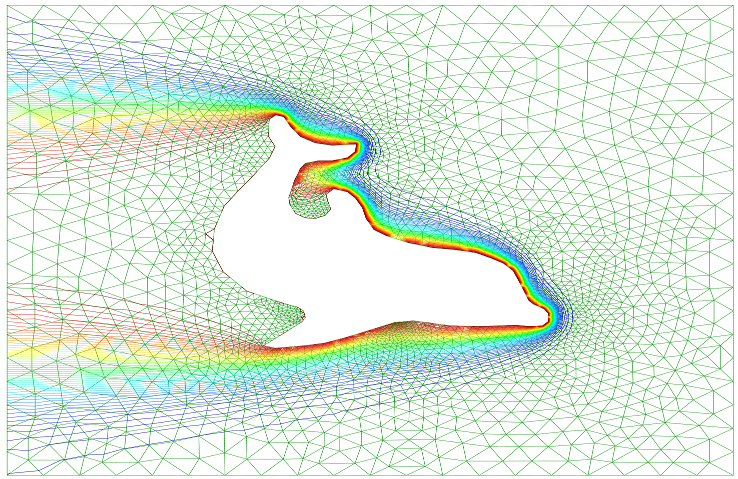
Convection–diffusion:

$$\dot{u} + b \cdot \nabla u - \nabla \cdot (\epsilon \nabla u) = f,$$

with $b = (-10, 0)$, $f = 0$ and $\epsilon = 0.1$ around a hot dolphin.

Mesh generation with **MATLAB** and visualization using *contour lines* in **GiD**.

MATLAB / GiD



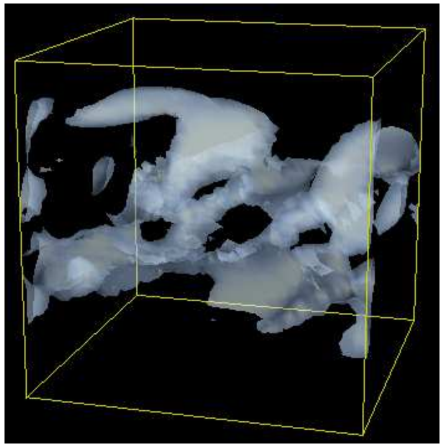
OpenDX

Incompressible Navier–Stokes:

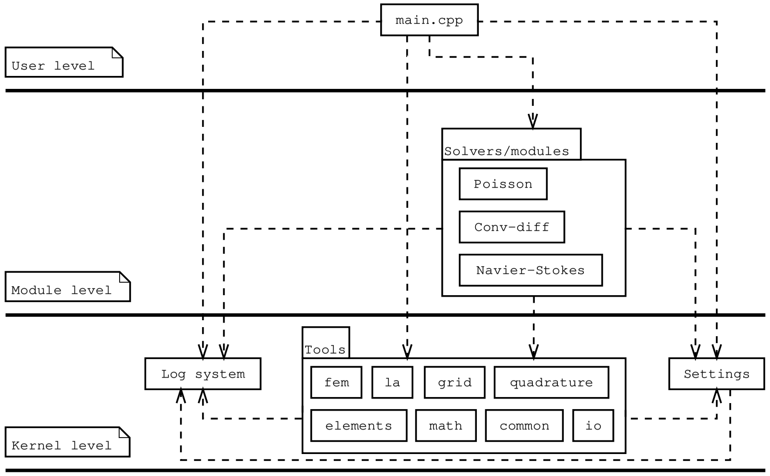
$$\begin{aligned} \dot{u} + u \cdot \nabla u - \nu \Delta u + \nabla p &= f, \\ \nabla \cdot u &= 0. \end{aligned}$$

Visualization in **OpenDX** of the isosurface for the velocity in a computation of transition to turbulence in shear flow on a mesh consisting of 1,600,000 tetrahedral elements.

OpenDX



Code structure



Using DOLFIN

Three levels

- Simple C/C++ interface for the *user* who just wants to solve an equation with specified geometry and boundary conditions.
- New algorithms are added at *module level* by the developer or advanced user.
- Core features are added at *kernel level*.

Solving Poisson's equation

```
int main()
{
  Mesh mesh("mesh.xml.gz");
  Problem poisson("poisson", mesh);

  poisson.set("source", f);
  poisson.set("boundary condition", mybc);

  poisson.solve();

  return 0;
}
```


Implementing a solver

```
void PoissonSolver::solve()
{
  Galerkin     fem;
  Matrix       A;
  Vector       x, b;
  Function     u(mesh, x);
  Function     f(mesh, "source");
  Poisson      poisson(f);
  KrylovSolver solver;
  File         file("poisson.m");

  fem.assemble(poisson, mesh, A, b);
  solver.solve(A, x, b);

  u.rename("u", "temperature");
  file << u;
}
```


Automatic assembling

```
class Poisson : public PDE {
  ...
  real lhs(const ShapeFunction& u, const ShapeFunction& v)
  {
    return (grad(u),grad(v)) * dK;
  }

  real rhs(const ShapeFunction& v)
  {
    return f*v * dK;
  }
  ...
};
```


Automatic assembling

```
class ConvDiff : public PDE {
  ...
  real lhs(const ShapeFunction& u, const ShapeFunction& v)
  {
    return (u*v + k*((b,grad(u))*v + a*(grad(u),grad(v))))*dK;
  }

  real rhs(const ShapeFunction& v)
  {
    return (up*v + k*f*v) * dK;
  }
  ...
};
```

Handling meshes

Basic concepts:

- `Mesh`
- `Node`, `Cell`, `Edge`, `Face`
- `Boundary`
- `MeshHierarchy`
- `NodeIterator`
  `CellIterator`
  `EdgeIterator`
  `FaceIterator`

Handling meshes

Reading and writing meshes:

```
File file(``mesh.xml'');
Mesh mesh;
file >> mesh; // Read mesh from file
file << mesh; // Save mesh to file
```


Handling meshes

Iteration over a mesh:

```
for (CellIterator c(mesh); !c.end(); ++c)
  for (NodeIterator n1(c); !n1.end(); ++n1)
    for (NodeIterator n2(n1); !n2.end(); ++n2)
      cout << *n2 << endl;
```


Linear algebra

Basic concepts:

- `Vector`
- `Matrix` (sparse, dense or generic)
- `KrylovSolver`
- `DirectSolver`

Linear algebra

Using the linear algebra:

```
int N = 100;
Matrix A(N,N);
Vector x(N);
Vector b(N);
b = 1.0;
for (int i = 0; i < N; i++) {
  A(i,i) = 2.0;
  if ( i > 0 )
    A(i,i-1) = -1.0;
  if ( i < (N-1) )
    A(i,i+1) = -1.0;
}
A.solve(x,b);
```


Summary of features

Summary of features

Implemented features:

- Automatic assembling
- Linear elements in 2D and 3D
- Adaptive mesh refinement
- Basic linear algebra
- Solvers for Poisson and convection–diffusion
- Log system
- Parameter management

Summary of features

In preparation:

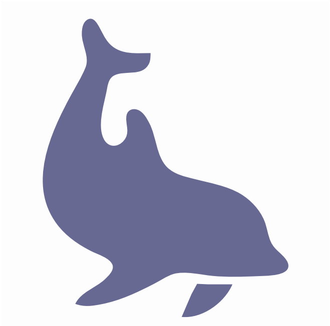
- Multi-adaptive ODE-solver (Jansson/Logg)
- Improved preconditioners (Hoffman/Svensson)
- Improved linear algebra (Hoffman/Logg)

Summary of features

Wishlist / help wanted:

- Multi-grid
- Implementation of boundary conditions
- Eigenvalue solvers
- Higher-order elements
- Documentation
- New solvers / modules
- Testing, bug fixes

Web page



- `www.phi.chalmers.se/dolfin`

Overview of Puffin

Introduction

- A simple and minimal version of DOLFIN
- Developed at the Department of Computational Mathematics
- Written for Octave/Matlab
- Licensed under the GNU GPL
- `http://www.fenics.org/puffin`
- Used in the computer sessions for the Body & Soul project

Using Puffin

Based around the two functions `AssembleMatrix()` and `AssembleVector()` that are used to assemple a linear system

$$AU = b,$$

representing a variational formulation

$$a(u, v; w) = l(v; w) \quad \forall v \in V,$$

where $a(u, v; w)$ is a bilinear form in u (the trial function) and v (the test function), and $l(v; w)$ is a linear form in v.

Using Puffin

A variational formulation is specified as follows:

```
function integral = MyForm(u, v, w, du, dv, dw, dx, ds, x, d, t, eq)
if eq == 1
  integral = ... * dx + ... * ds;
else
  integral = ... * dx + ... * ds;
end
```


Using Puffin

Example: Poissons equation.

$$\int_\Omega \nabla u \cdot \nabla v \, dx + \int_\Gamma \gamma u v \, ds = \int_\Omega f v \, dx + \int_\Gamma (\gamma g_D - g_N) v \, ds \quad \forall v.$$

```
function integral = Poisson(u, v, w, du, dv, dw, dx, ds, x, d, t, eq)

if eq == 1
  integral = du'*dv*dx + g(x,d,t)*u*v*ds;
else
  integral = f(x,d,t)*v*dx + (g(x,d,t)*gd(x,d,t) - gn(x,d,t))*v*ds;
end
```


Using Puffin

Syntax of the function `AssembleMatrix()`:

```
A = AssembleMatrix(points, edges, triangles, pde, W, time)
```

Syntax of the function `AssembleVector()`:

```
b = AssembleVector(points, edges, triangles, pde, W, time)
```

Web page

- http://www.fenics.org/puffin