PDE Project Course

Introduction

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Partial differential equations (PDE)

Solve

$$A(u) = f$$

where A is a differential operator, f is a given force term and u is the solution.

Important questions:

- Existence of solutions
- Uniqueness of solutions
- Computation of solutions

Examples of PDE

- $A(u) = -\Delta u = f$ Poisson's equation
- $A(u) = \dot{u} - \Delta u = f$ The heat equation
- $A(u) = \ddot{u} - \Delta u = f$ The wave equation

These are the main examples of linear *elliptic*, *parabolic*, and *hyperbolic* PDE.

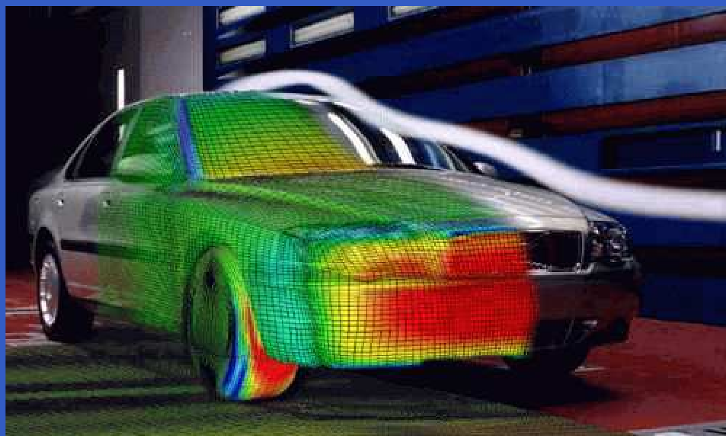
Examples of PDE

The Navier–Stokes equations:

$$A(u) = \begin{pmatrix} \dot{v} + v \cdot \nabla v - \nu \Delta v + \nabla p \\ \nabla \cdot v \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}$$

where the solution $u = (v, p)$ consists of the the fluid velocity v and the pressure p.

Examples of PDE



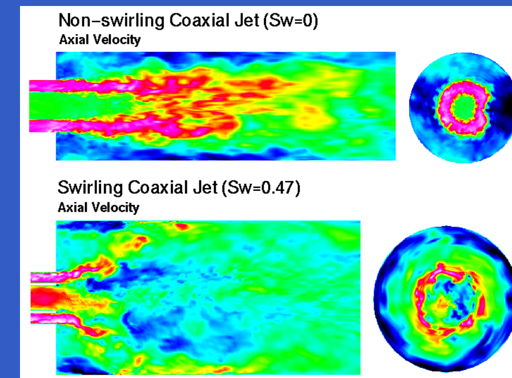
Numerical solution of the Navier-Stokes equations.

Examples of PDE

Systems of convection–diffusion–reaction equations:

$$A(u) = \begin{pmatrix} \dot{u}_1 + \beta \cdot \nabla u_1 - \Delta u_1 + r_1(u) \\ \dots \\ \dot{u}_n + \beta \cdot \nabla u_n - \Delta u_n + r_n(u) \end{pmatrix} = \begin{pmatrix} f_1 \\ \\ f_n \end{pmatrix}$$

Examples of PDE



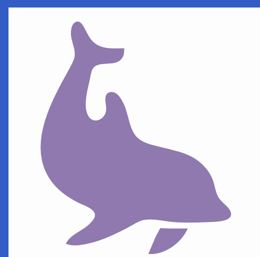
Numerical solution of convection-diffusion-reaction equations: fuel injection in an engine.

Lecture plan

1. *Adaptive finite element methods*
2. *Implementation of adaptive finite element methods*
3. *C++ programming*
4. *An introduction to DOLFIN and Puffin*

DOLFIN

- Numerical solution of PDE using FEM
- 2D / 3D
- Object-oriented (C++)
- GPL



Puffin

- Numerical solution of PDE using FEM
- 2D
- Written for Matlab (Octave)
- GPL

Project ideas

- Simulation / mathematical modelling
- Mesh generation
- Algebraic solvers
- Vizualisation
- Code optimization
- . . .

Course web page

Follow the links education → courses, starting at

- `http://www.math.chalmers.se/cm/`

My homepage:

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