D. Example Application

We have a container with water of temperature 10 degrees, which we wish to heat to an average temperature of 40 degrees by inserting the container into boiling water (100 degrees). The surrounding water remains boiling throughout the heating process. The shortest side of the container is 5 cm and we may assume the other two sides are much longer. The heat conduction coefficient a is 1.

a: Formulate a suitable 1D heat equation which models the heating process. Start from the one dimensional time dependent model problem and identify the proper coefficients.

b: Unfortunately we do not know the permeability coefficient γ. To determine γ we have access to a measurement of the temperature distribution at time $T = 0.5$ minutes at certain values of x given in the following table.

x	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50
u	15.50	13.34	11.55	10.93	10.63	11.16	11.53	13.42	15.95

Use these data to estimate the value of the permeability γ. Note that you should not expect to get a perfect match with the measurements, since there are always errors in the measurements (and also in your numerical simulations). Present a plot of your results.

c: Use your knowledge of γ to estimate at what time you should stop the heating process to get the desired average temperature 40 degrees.

d: If you happen to forget that you are heating water and let the process go on for too long a time. What is the result ? Think first and then verify using your solver.

Comment: The situation in this problem is typical for many applications: some of the coefficients in the differential equation modeling a physical process are not known. Typically, we can find values of some of the coefficients in, for instance a handbook, and we need to determine some coefficients using measurements. The latter situation leads to an optimization problem where we seek coefficients which produce solutions which match the measurements as well as possible. Such problems are called inverse problems.