Complementary exercises on ordinary differential equations

1. Solve the following initial value problems (hint: integrating factor)

$$\begin{array}{lll} (a) & u'(x) - 4u(x) = 0, & u(0) = 1 \\ (b) & y'(x) + 2y(x) = 1, & y(0) = 2 \\ (c) & \dot{u}(t) = -3u(t) + 5, & u(1) = 1 \end{array}$$

2. Solve the following initial value problems (hint: integrating factor)

$$\begin{array}{lll} (a) & xy'(x) + 3y(x) = x^2, & y(2) = 1 \\ (b) & y'(x) = y(x) - xe^{-x}, & y(0) = 2 \end{array}$$

3. Solve the following initial value problems

$$\begin{array}{ll} (a) & (x^2+4)u'(x) - 4xu(x) = x, \quad u(1) = 1 \\ (b) & \cos^2 t\, \dot{y}(t) + y(t) = 0, \quad y(0) = 1 \\ (c) & (x+1)y'(x) + 2y(x) = (1+x)^2, \quad y(0) = 0 \end{array}$$

4. Solve the following initial value problems

$$\begin{array}{ll} (a) & u'(x) = \frac{1}{2}(x + u(x)), \quad u(1) = 1 \\ (b) & xu'(x) - u(x) = x\sin x, \quad u(\pi) = 0 \\ (c) & \dfrac{dy}{dx} + 2xy = xe^{-x^2}, \quad y(0) = 0 \end{array}$$

Answers:

$$\begin{array}{lll} \mathbf{1}. & (a) & u(x) = e^{4x} \quad (b) \quad \tfrac{1}{2}(3e^{-2x}+1) \quad (c) \quad \tfrac{1}{3}(5 - 2e^{-3t+3}) \\ \mathbf{2}. & (a) & y(x) = \dfrac{x^5+8}{5x^3} \quad (b) \quad \tfrac{1}{4}(1 + 7e^{2x} + 2x)e^{-x} \\ \mathbf{3}. & (a) & u(x) = \tfrac{1}{20}(11 + 8x^2 + x^4) \quad (b) \quad y(t) = e^{-\tan x} \\ & (c) & y(x) = \dfrac{4x + 6x^2 + 4x^3 + x^4}{4(x+1)^2} \\ \mathbf{4}. & (a) & 4e^{-1/2+x/2} - x - 2 \quad (b) \quad x\displaystyle\int_\pi^x \frac{\sin s}{s}\, ds \quad (c) \quad \frac{x^2}{2}e^{-x^2} \end{array}$$