COMPLEMENTARY INTEGRATION EXERCISES TO AMBS PART II

1. Compute (hint: integrate by parts)

$$(a)\quad \int_0^\infty xe^{-x}\,dx \quad (b)\quad \int_1^2 \sqrt{x}\ln x\,dx \quad (c)\quad \int_0^\pi x\sin x\,dx$$

2. Compute (hint: integrate by parts)

$$(a)\quad \int_1^4 \frac{e^{1/x}}{x^3}\,dx \quad (b)\quad \int_1^2 (\ln x)^2\,dx \quad (c)\quad \int_0^t x^3 e^{-x^2}\,dx$$

3. Compute

$$(a)\quad \int_0^1 x\sqrt{1+x}\,dx \quad (b)\quad \int_{-a}^a x^3\cos(ax)\,dx \quad (c)\quad \int_1^2 \ln(ax)\,dx$$

4. Compute (hint for (a) : $3^x = e^{x\ln 3}$)

$$(a)\quad \int_0^1 x\cdot 3^x\,dx \quad (b)\quad \int_{-1}^0 x(1+x)^9\,dx \quad (c)\quad \int_0^{2\pi} e^x\sin x\,dx$$

5. Compute (hint: change of variable, assume n positive integer)

$$(a)\quad \int_0^1 \frac{x^2}{1+x^3}\,dx \quad (b)\quad \int_0^t \frac{x^{n-1}}{1+x^n}\,dx \quad (c)\quad \int_1^2 \frac{2x+1}{x^2+x}\,dx$$

6. Compute

$$(a)\quad \int_0^\pi \sin^2 x\,dx \quad (b)\quad \int_0^{2\pi} \cos^2 x\,dx \quad (c)\quad \int_0^\pi \sin x\cos x\,dx$$

(hint: $\sin^2 x = \frac{1}{2}(1-\cos 2x)\,,\quad \cos^2 x = \frac{1}{2}(1+\cos 2x)\,,\quad \sin x\cos x = \frac{1}{2}\sin 2x$)

Answers:

1. $(a)\quad 1 \quad (b)\quad \frac{4}{9}(1-2\sqrt{2}+3\sqrt{2}\ln 2) \quad (c)\quad \pi$

2. $(a)\quad \dfrac{3e^{1/4}}{4} \quad (b)\quad 2(1-2\ln 2+(\ln 2)^2) \quad (c)\quad \frac{1}{2}(1-e^{-t^2}(1+t^2))$

3. $(a)\quad \frac{4}{15}(1+\sqrt{2}) \quad (b)\quad 0 \quad (c)\quad 2\ln 2a - \ln a - 1$

4. $(a)\quad (3\ln 3-2)/(\ln 3)^2 \quad (b)\quad -1/110 \quad (c)\quad \frac{1}{2}(1-e^{2\pi})$

5. $(a)\quad (\ln 2)/3 \quad (b)\quad \ln(1+t^n)/n \quad (c)\quad \ln 3$

6. $(a)\quad \pi/2 \quad (b)\quad \pi \quad (c)\quad 0$