Övningstentamen i

Analys och Linjär Algebra del A

för Kf/Kb lp I - 00/01

Datum:	11/10 – 2000
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Hjälpmedel:	Penna, sudd och linjal
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1. (a) Derive the chain rule, i.e. compute the derivative of the function $f \circ g$, where f and g are two differentiable functions.

   (b) Compute $D((f \circ g)^{-1})$ for $f(x) = x^2$ and $g(x) = x + 5$. Assume $x \geq -5$.

2. (a) Let $\{x_n\} = \{0.1, 0.13, 0.135, 0.1357, 0.13579, 0.135791, \dots\}$. Is this a Cauchy sequence?

   (b) Show that $\lim_{n\to\infty} \frac{n^2+2}{n^2+1} = 1$ by using the formal definition.

   (c) Find the value of $1 + 1/3 + 1/9 + 1/27 + \dots$.

3. (a) Let $R(\theta)$ be the matrix for rotating a vector the angle θ. Show that $[R(\theta)]^2 = R(2\theta)$, using the formulas

$$\begin{aligned} \sin 2\theta &= 2 \sin\theta\cos\theta, \\ \cos 2\theta &= \cos^2\theta - \sin^2\theta. \end{aligned} \tag{1}$$

   (b) Is it true that $[R(\theta)]^n = R(n\theta)$? Why? Why not?

   (c) Derive expressions for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ in terms of $\sin\alpha, \sin\beta, \cos\alpha, \cos\beta$. Hint: Continue your discussion from b).

   (d) Without doing any computations, compute $[R(\theta)]^6$ and $[R(\theta)]^{12}$ for $\theta = 30°$.

4. (a) Define the concept of Lipschitz continuity.

   (b) Is the function $f(x) = \frac{1}{x^2-4}$ Lipschitz continuous on $(2, 3]$? Is it Lipschitz continuous on $[5, 9]$? If so, compute the Lipschitz constant.

   (c) Show that $f(x) = \sqrt{x^2+2}$ is Lipschitz continuous on $[1, 2]$ and compute the Lipschitz constant.

5. (a) Show that Newton's method converges quadratically for a simple root.

   (b) Do one iteration with Newton's method for solving the system of equations

$$\begin{aligned} x_1 + x_2^2 + 4 &= 0 \\ \sin x_1 + x_2 - 2 &= 0 \end{aligned}$$

   with $x^0 = (0, 0)$.

6. Let x_i be the sequence generated by the contraction mapping g as $x_{i+1} = g(x_i)$.

   (a) Show that $x = \lim_{i\to\infty} x_i$ is a fixpoint.

   (b) Determine the smallest possible value of a so that $g(x) = \frac{1}{2x} + 5$ is a contraction on $[1, a]$.