ALA-a - project 1 - application areas

Geometry: Volumes

1. Consider a conical water reservoir with a cylindrical elevator schact as in the figure. Find the volume of water in the reservoir as a function of the height of water. When is the reservoir half filled?

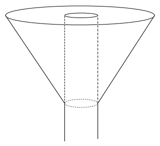

Figure 1:

2. To be able to design a new ventilation system of the Stockholm globe, find its volume. Given its volume, determine the diameter of the building.

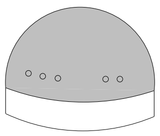

Figure 2:

3. Consider a cylindrical oil reservoir of diameter 2 and length 1. The volume of oil in the reservoir is given by the formula $V = f(y) = y\sqrt{1-y^2} + \arcsin(y) + \pi/2$, where y is the vertical coordinate, and $\theta = \arcsin(y)$ is the angle (in radians in the interval $[-\pi/2, \pi/2]$) for which $\sin(\theta) = y$. At what level y is the reservoir filled to one quarter?

4. Design a hot air ballon of a fancy geometrical form (to get sponsoring to actually build the ballon) with a given volume (sufficient to obtain take off with a certain load).

Mechanics: Spring systems

1. Find the spring constant $k = f(k_1, k_2)$ of a spring that can replace two springs with spring constants k_1 and k_2 coupled a) in parallell b) series.

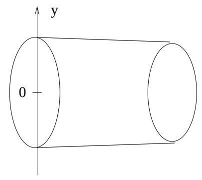

Figure 3:

2. Design more complex spring systems by succesively adding new springs in parallell or series to the basic systems in 1a,b, systematically or more random. Find the equivalent spring that can replace the system.

3. Seek the spring constant of a particular spring in the system given the other spring constants and the spring constant of the system.

Electronics: Resistor systems

1. Find the resulting resistance $r = f(r_1, r_2)$ of two resistors of r_1 and r_2 (Ohms) resp., coupled a) in series b) in parallell.

Figure 4:

2. Consider more complex resistor systems by succesively adding new resistors to the basic systems in 1a,b, systematically or more random. In each case, find the resulting resistance of the system.

3. Seek the resistance of a particular resistor in the system given the resistances of the other resistors and the resistance of the system as a whole.

Population Dynamics: Rabbits and Bacteria

1. Study the model Changes in a population of insects in Chap 5 and solve Problems 5.4, 5.5 and maybe also 5.6. Study the model Growth of bacteria in Chap 6.

2. Construct similar models yourself expressing the population at year n in terms of the population at year $n-1$ (and possibly also year $n-2$ et cet).

Take positive and negative factors to population growth into account. Try to find simple closed form expressions for the the size of the population after n steps in terms of the initial population. Is this always successful? What happens if you can't find that neat formula?

3. Consider a parameter in the model (such as R and K in the insect and bacteria models above), as an unknown and solve the equation of finding the value of the unknown parameter that produces a certain population after a certain number of years.

4. Construct models for several interacting populations. Solve some related equation considering one of the parameters involved to be unknown.

5. Explain the phenomena of "lämmelår".

Economy: Bank accounts and Stock market.

1. Do the same as in Project Population Dynamics with the interpretation of a capital of money instead of a population of rabbits. Remember that $(1+\frac{x}{n})^n$ is the capital after n capitalizations of size $\frac{x}{n}$. Model capital growth taking postive and negative factors into account.

2. Solve equations like $(1+\frac{x}{n})^n = K$ in x to find for instance the interest rate which gives a certain capital growth after a certain number of capitalizations.

3. Construct models for several interacting capitals of money (companies, stocks,funds et cet). Solve some related equation considering one of the parameters involved to be unknown.

4. Explain the phenomena of Hausse, Baisse or Crash of the stock market.

Music: What is the sound of $2^{1/12}$?

Ett piano har en sk tempererad stämning där kvoten mellan frekvenserna för två närliggande halvtoner alltid är $2^{1/12}$. En oktav har 12 olika halvtoner.

Den Pythagoreiska durskalan i C (med tonerna CDEFGAHC) har följande frekvenskvoter: 1, 9/8, 81/64, 4/3, 3/2, 27/16, 243/128, 2.

Den sk "rena stämningen" för en durskala har frekvenskvoterna 1, 9/8, 5/4, 4/3, 3/2, 5/3, 15/8, 2. (Motsvarande interval kallas sekund, ters, kvart, kvint, sext, och stor septima.)

1. Bestäm den procentuella frekvensskillnaden för alla tonerna i en durskala mellan (i) tempererad stämning och Pythagoreisk stämning, (ii) tempererad stämning och "ren stämning" .

2. Förklara principerna bakom de olika stämningarna.

3. Upprepade kvintsprång CGDAEHFissCissAssEssBbFC ger skalans alla toner. Förklara varför man skulle vilja att $(3/2)^{12} = 2^m$ för lämpligt naturligt tal m. Gäller detta?

4. Förklara varför ett piano med tempererad stämning kan låta bra tillsammans med fiol som är stämd i rena kvinter.

5. Om Du vill reducera frekvensskillnaden till säg hälften i tempererad stämning med n tonsteg per oktav jämfört med Pythagoreisk eller ren stämning, hur stort borde då n vara?

6. Utvidga resonemanget till olika mollskalor.

Hjälpmedel: Matlab som kalkylator för rationella tal (add, subtr, mult, div).