ALA-a KfKb - 'project 2'

Pass the Exam

Each tutor group formulates problems for (half of) a written exam for ALA-a. Groups with odd/even numbers formulate problems from sections with odd/even numbers below. The groups with odd numbers then exchange exams with the groups with even numbers, so that groups which have formulated problems from odd numbers sections take an exam based on even numbers sections on Wednesday, and vice versa.

Fundamental material Analysis A Kf/Kb00

As a background we list below the fundamental basic topics of Analysis A. Each student should be fully familiar with each one of these topics, which form the basic core of Analysis A.

1. Rational, real and complex numbers

Formulate and prove the basic rules for operating with rational, real and complex numbers, given the basic rules for integers. Give basic examples including features such as parentheses, double fractions, $(a+b)^2$, $(a+b)^3$, completing squares, and solving second degree polynomial equations (including such with complex coefficients), and of the form $z^n = a$ with n a natural number. State and prove the formula for a finite geometric series.

2. Functions

What is a function $f(x)$ of x? Explain the use of the notation $f : D(f) \to R(f)$. How can functions be combined (addition, multiplication, division, composition, inverse)? Give examples of functions, $D(f)$, $R(f)$, and combinations.

3. Sequences

Define the concept of a sequence of numbers $\{x_i\}_{i=1}^{\infty}$. Define the concepts of converging sequence and Cauchy sequence. Prove that a Cauchy sequence of real numbers converges to a real number.

4. Lipschitz continuity

Define Lipschitz continuity. Determine the Lipschitz constant for some polynomials or rational functions on bounded intervals. Determine Lipschitz constants for combinations of functions arising from linear combinations, products, quotients, and compositions. Prove that if $f(x)$ is Lipschitz continuous, then

$$\lim_{i\to\infty} f(x_i) = f(\lim_{i\to\infty} x_i)$$

5. Intermediate Value Theorem

Prove the Intermediate Value theorem. Define the function x^r for r a rational constant and $x > 0$, and derive basic computational rules like $x^r x^s = x^{r+s}$.

6. Contraction Mapping theorem

Prove the Contraction Mapping theorem very carefully with all details.

7. Linear algebra - vectors and matrices

Define the fundamental algebraic operations involving vectors and matrices including addition, multiplication by a scalar ('scaling'), and matrix multiplication. Define the scalar product of vectors, and the projection of a vector onto another vector or line. Also, give geometric interpretations and some application. Express the geometric operations a) rotation (clockwise a given angle θ, say , b) translation (in the direction of $(3, 4)$ a distance 2, for example, c) reflection (in the line $2x_1 = x_2$, say) and d) projection of vectors (onto the same line) x as matrix-vector multiplications Ax with suitable matrices A. Explain the relation of (the matrix/vector equation) $Ax = b$ to $\sum x_i a_i = b$, and to the corresponding system of linear equations, where a_i are the columns of A. Explain how to solve a given linear system of equations, and how to compute an inverse matrix, by example.

8. Derivative

Define the derivative of a function $f(x)$. Prove that $Dx^2 = 2x$, $Dx^3 = 3x^2$, ... $Dx^n = nx^{n-1}$, $Dx^r = rx^{r-1}$, r rational.

9. Differentiation rules

State and prove the fundamental rules for computing with derivatives including linear combinations, products, quotients, chain rule, derivative of inverse, implicit differentiation. Give concrete examples of application of these rules.

10. Newton's method

Analyze convergence of Newton's method in the case of a single root (double root optional).

11. l'Hopital's rule

State and prove l'Hopital's rule.

12. Basic differential equation models

Present basic models leading to fundamental differential equations of the form $u'(x) = f(x)$, $u'(x) = u(x)$, $u''(x) + u(x) = 0$.