EXAMPLES OF POSSIBLE EXAM QUESTIONS FOR THE TMA 482 COURSE (PDE SC TM 2003)

1. Prove an a posteriori error estimate using duality for the backward Euler method (dG(0)) for a system of ODEs. Formulate a corresponding adaptive algorithm.

2. Formulate basic error estimates for a finite element method for the heat equation. Indicate the basic principles in the proofs.

3. Formulate and prove basic properties of scalar conservation laws: characteristics, Rankine-Hugoniot weak solutions, entropy inequality.

   The Rankine-Hugoniot condition relates the shock speed s to the jumps in u and $f(u)$:

$$s = \frac{[f(u)]}{[u]}$$

4. Prove strong stability estimates for the parabolic model problem:

$$\begin{aligned} \dot{u} + Au &= 0,\ t > 0 \\ u(0) &= u^0, \end{aligned}$$

   where A is a positive semi-definite n x n-matrix (hint: multiply by u, tAu, t^2A^2u).

5. Prove a strong stability estimate for the dual linearized Burgers equation in the case of a viscous shock.

$$(-\dot{\phi} + a\phi_x - \epsilon\phi_{xx} = 0,\ a_x \leq 0)$$

6. Prove an a posteriori error estimate for shock solution of Burgers equation for an algorithm of type "exact transport + projection".

7. Prove a basic energy estimate for incompressible Navier-Stokes equations (hint: multiply by u).

8. Discuss stability properties of the linearlized Navier-Stokes equations in the case of Couette flow.

9. Discuss the possibility of explicit time-stepping for parabolic (stiff) problems.

10. Present the basic principles for optimal control of systems described by ordinary differential equations: objective function, Lagrangean formulation and optimality criteria. Present a simple concrete case.

11. Formulate duals for some given ODEs and discuss stability properties based on the duals.